Beyond the Point Particle: LES-Style Filtering of Finite-Sized Particles

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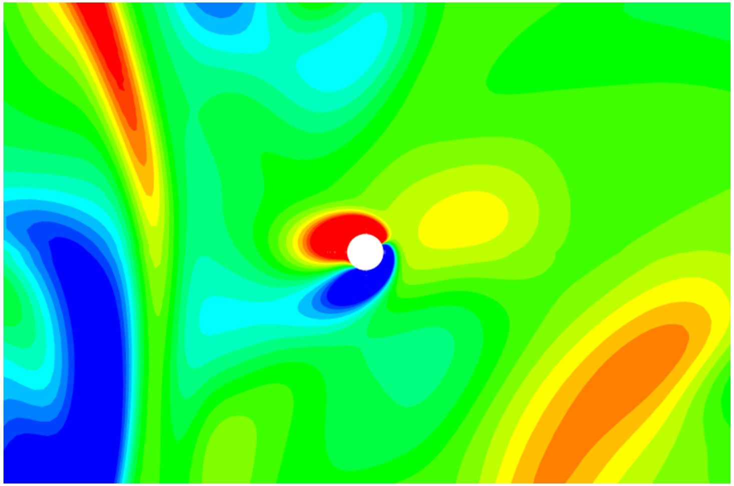
Motivation

- Small particles in multiphase turbulent flows:



Sphere in turbulent flow, Tristan Burton, 2003.

- Most LES calculations treat small particles as point forces.
  - Is a point force a sufficiently accurate model?
  - How can point-force models be improved?
- Objective: Derive particle models without assuming the point-particle limit, in a manner fundamentally consistent with LES filtering.

Navier-Stokes on an Embedded Subdomain

- Mass and momentum conservation equations for an embedded subdomain containing the fluid phase:

$$\frac{\partial(\phi_i \rho_i)}{\partial t} + \nabla \cdot (\phi_i \rho_i \vec{\boldsymbol{u}}_i) = \rho_i \vec{\boldsymbol{u}}_{\text{outflow},\ i} \cdot \nabla \phi_i$$

$$\frac{\partial(\phi_i \rho_i \vec{\boldsymbol{u}}_i)}{\partial t} + \nabla \cdot (\phi_i \rho_i \vec{\boldsymbol{u}}_i \vec{\boldsymbol{u}}_i) = -\nabla(\phi_i p_i) + \nabla \cdot (\phi_i \vec{\vec{\boldsymbol{\tau}}}_i) + \vec{\vec{\boldsymbol{\sigma}}}_{\text{surface},\ i} \cdot \nabla \phi_i + \rho_i \vec{\boldsymbol{u}}_i \vec{\boldsymbol{u}}_{\text{outflow},\ i} \cdot \nabla \phi_i$$

- Phase function ϕ_i is 1 inside the fluid subdomain, and zero outside it. Thus, $\nabla \phi_i$ is a delta-function distribution along the subdomain boundary.
- This formulation is valid inside and outside the fluid phase, and contains both the internal field equations and the surface conditions for the fluid.
- Thus, it can be filtered across the fluid boundary in a mathematically rigorous manner, without requiring any special treatment for the boundary.

Spatially-Filtered Multiphase Flow Equations

- Filtering these equations (with $(\phi_i \rho_i)$, $(\phi_i \rho_i \vec{\boldsymbol{u}}_i)$, and $(\phi_i p_i)$ chosen as the primary variables) produces:

$$\frac{\partial \overline{(\phi_i \rho_i)}}{\partial t} + \nabla \cdot \overline{(\phi_i \rho_i \vec{\boldsymbol{u}}_i)} = \overline{\rho_i \vec{\boldsymbol{u}}_{\text{outflow, } i} \cdot \nabla \phi_i}$$

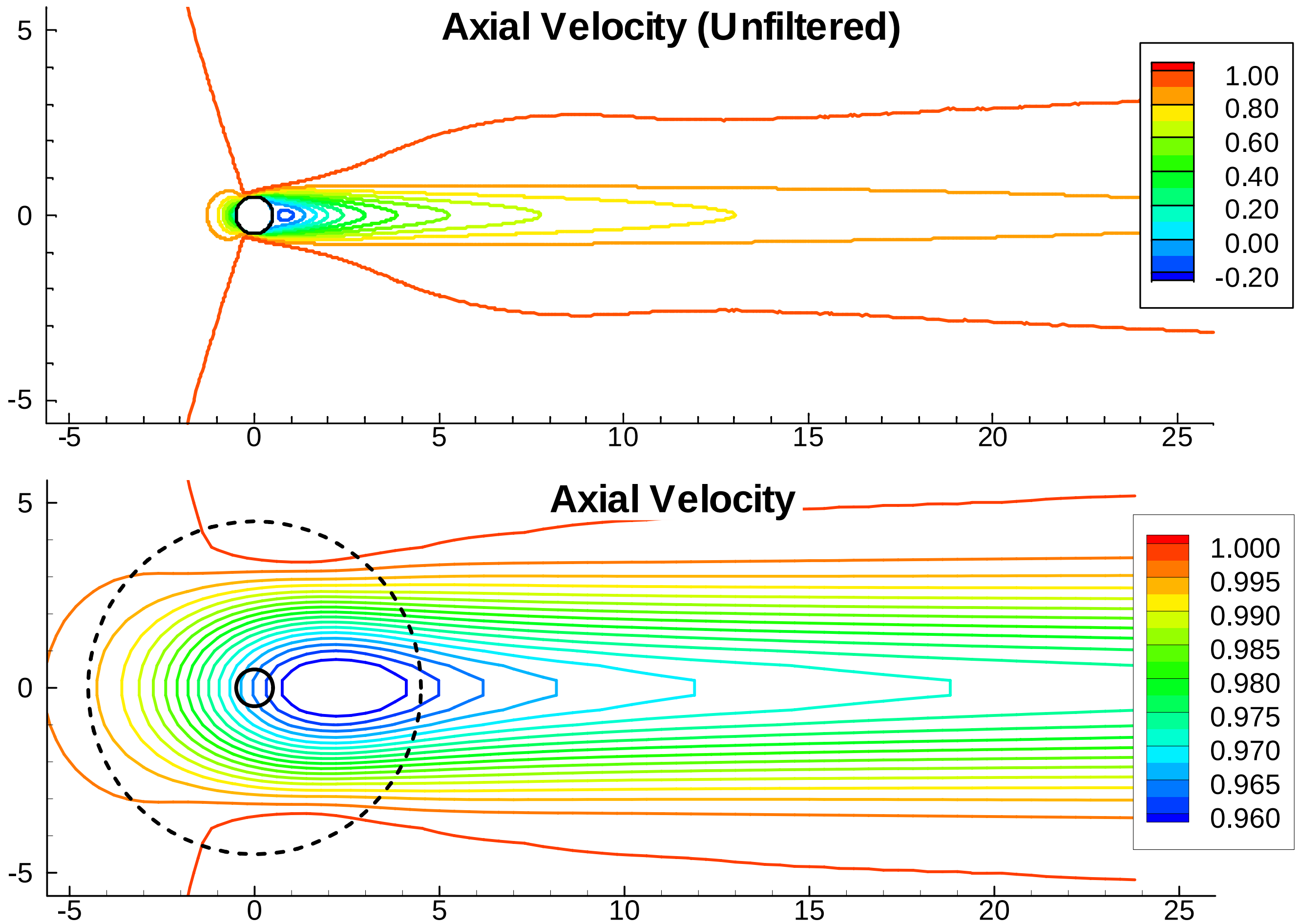
$$\frac{\partial \overline{(\phi_i \rho_i \vec{\boldsymbol{u}}_i)}}{\partial t} + \nabla \cdot \overbrace{(\phi_i \rho_i \vec{\boldsymbol{u}}_i \vec{\boldsymbol{u}}_i)}^{\text{resolved part}}$$
$$= -\nabla \overline{(\phi_i p_i)} + \nabla \cdot \overbrace{(\phi_i \vec{\vec{\boldsymbol{\tau}}}_i)}^{\text{resolved part}} - \vec{\boldsymbol{T}}_{\text{SFS}} + \vec{\boldsymbol{T}}_{\text{SFS-viscous}}$$
$$+ \overline{\vec{\vec{\boldsymbol{\sigma}}}_{\text{surface, } i} \cdot \nabla \phi} + \overline{\rho_i \vec{\boldsymbol{u}}_i \vec{\boldsymbol{u}}_{\text{outflow, } i} \cdot \nabla \phi_i}$$

- These contain unclosed terms, which are provided by various models (particle models, wall models, turbulence models, phase-interaction models, etc.):
  - (blue) Mass and momentum fluxes across the surface.
  - (purple) Subfilter-scale convection term (as in LES).
  - (green) Subfilter-scale viscous term.

Particle Model Development Process

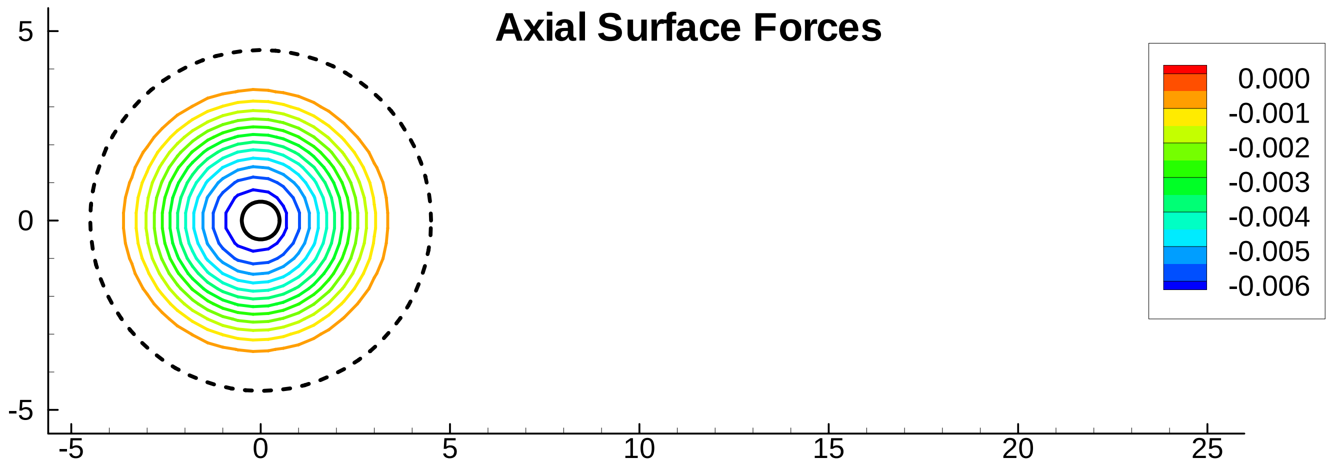
- Models for the unclosed terms can be developed by the following process:
  - Start with well-resolved calculations of representative flowfields around a particle.
  - Filter the resolved flowfields to the desired resolution.
  - Calculate exact forms of the subfilter-scale terms using fluxes from the filtered well-resolved flowfields.
  - Approximate the subfilter-scale terms with a parameterized model.
- Example application: Consider a single spherical particle in a steady uniform flow at Re=100.
- For this study, we use a grid much smaller than the filter size, decoupling the results from numerical effects.

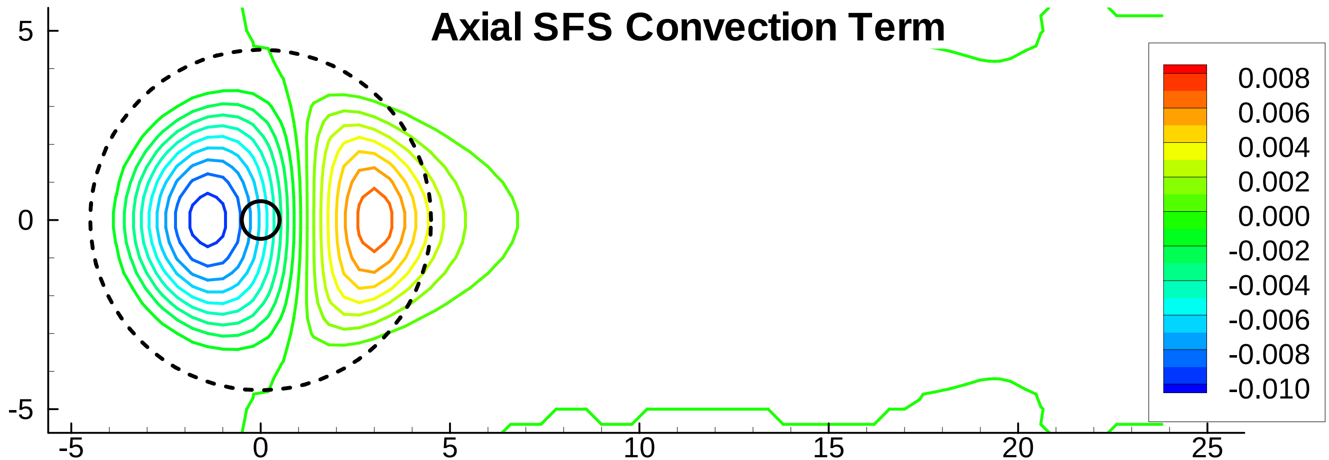
Exact Flow, $r_{\text{filter}} = 4.0D_{\text{particle}}$



- A “smallish” particle; nearly all of the flow details are washed out by the filter.

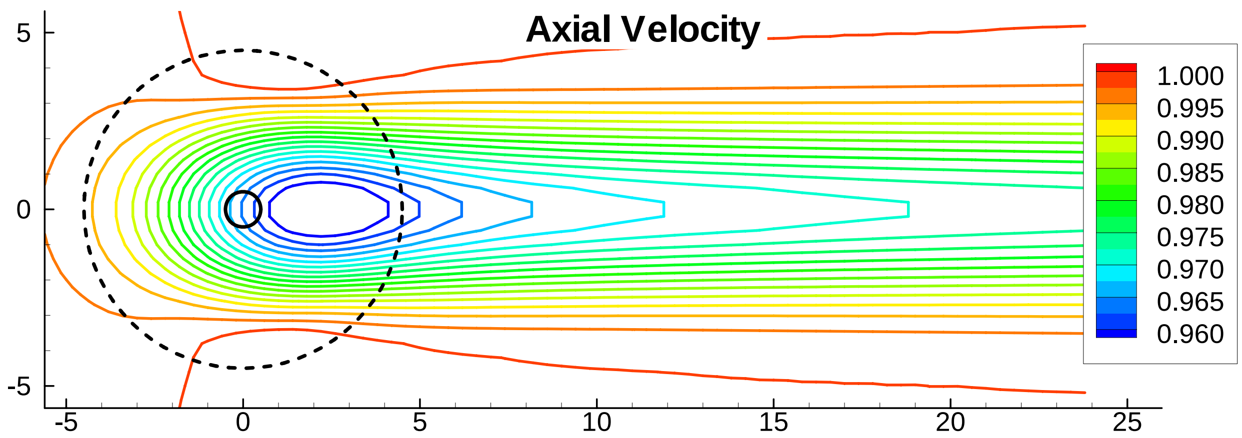
Exact Flow, $r_{\text{filter}} = 4.0 D_{\text{particle}}$

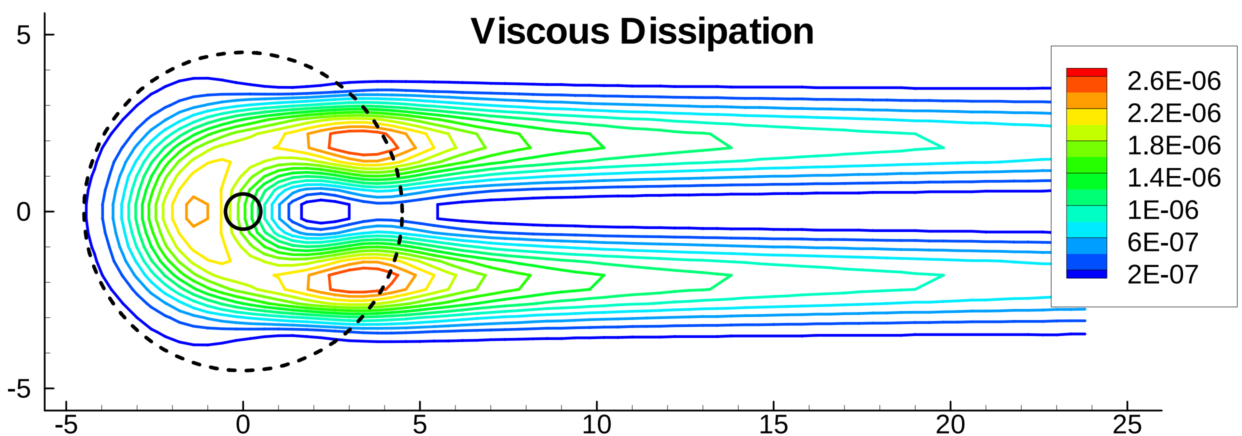




- Surface, SFS forces comparable in magnitude.
- The simplest nonzero SFS model is a point dipole.

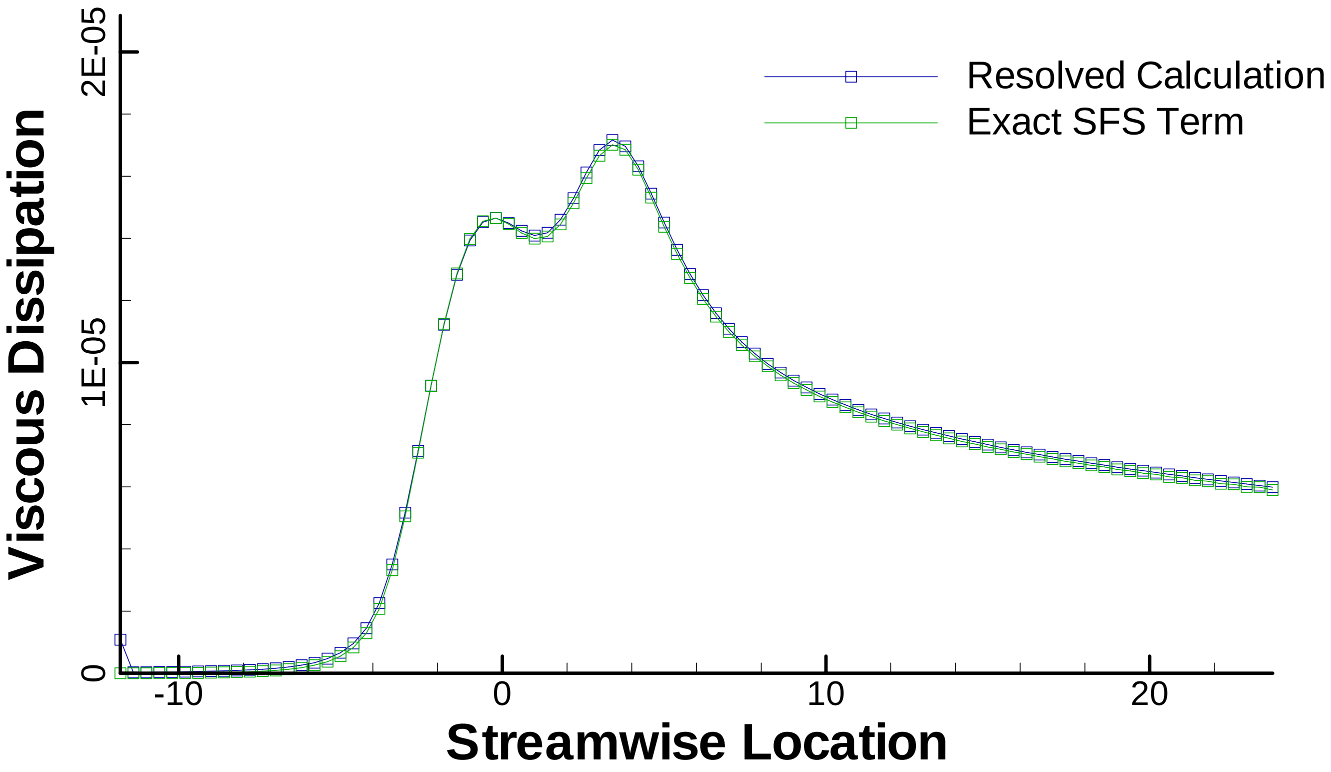
Exact Flow, $r_{\text{filter}} = 4.0 D_{\text{particle}}$





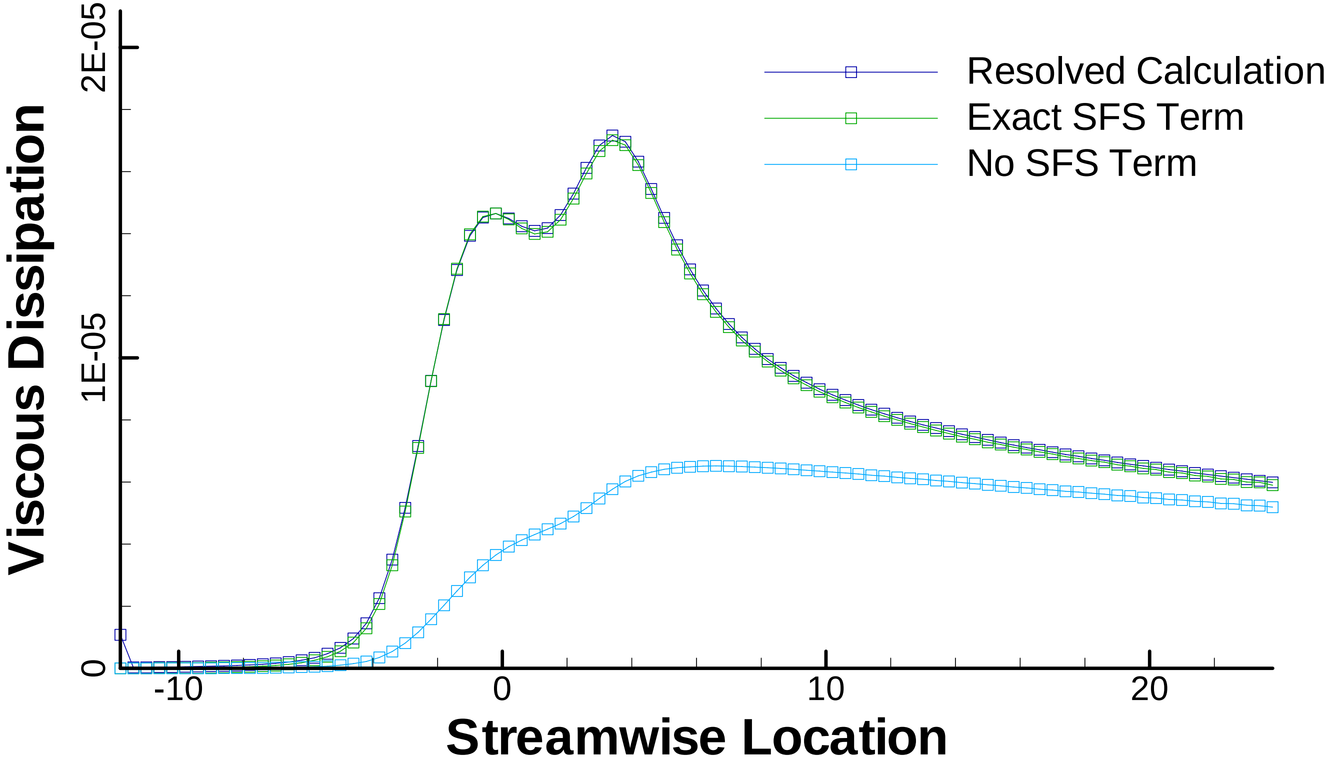
- The viscous dissipation will be used as a quantitative measure of the accuracy of computations with approximate models.

Results: $r_{\mathbf{filter}} = 4.0D_{\mathbf{particle}}$



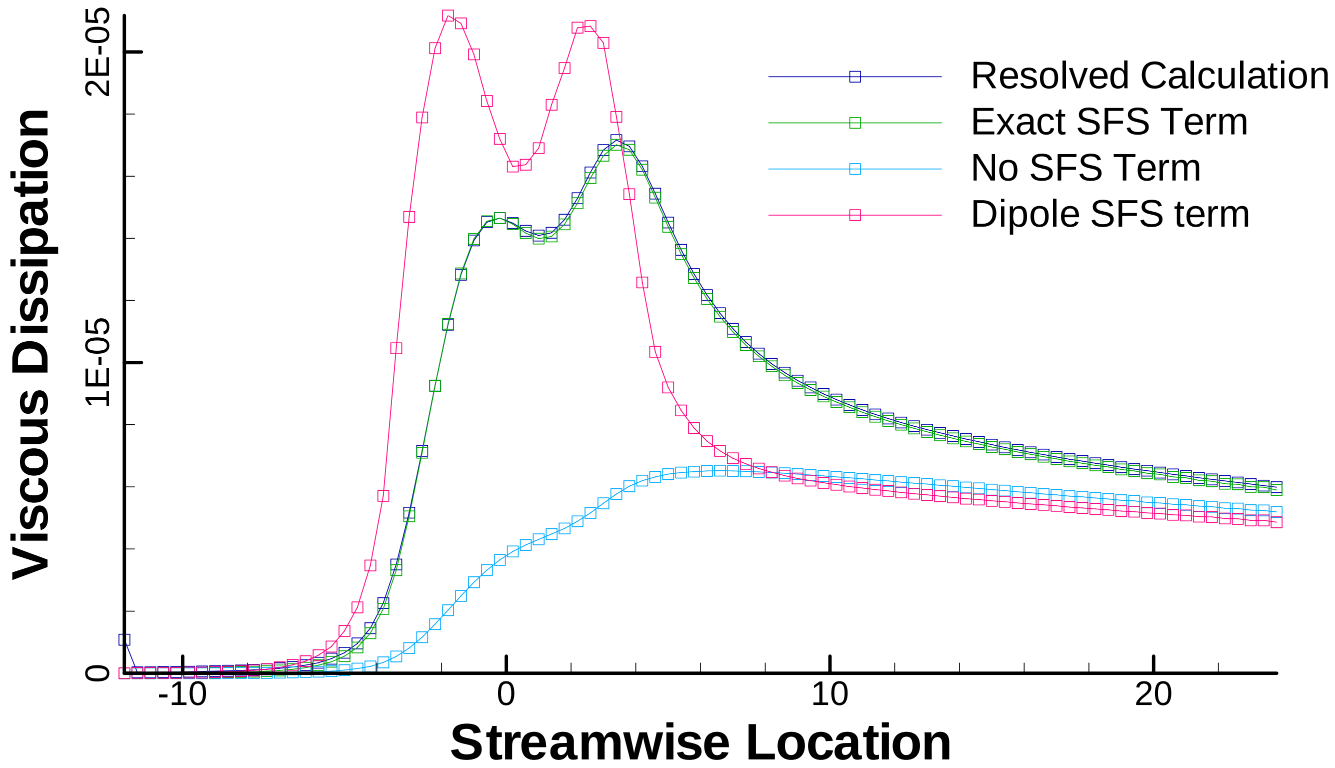
- With an exact force model, we recover the true flowfield, as expected.
- This validates the underlying numerical methods.

Results: $r_{\textbf{filter}} = 4.0D_{\textbf{particle}}$



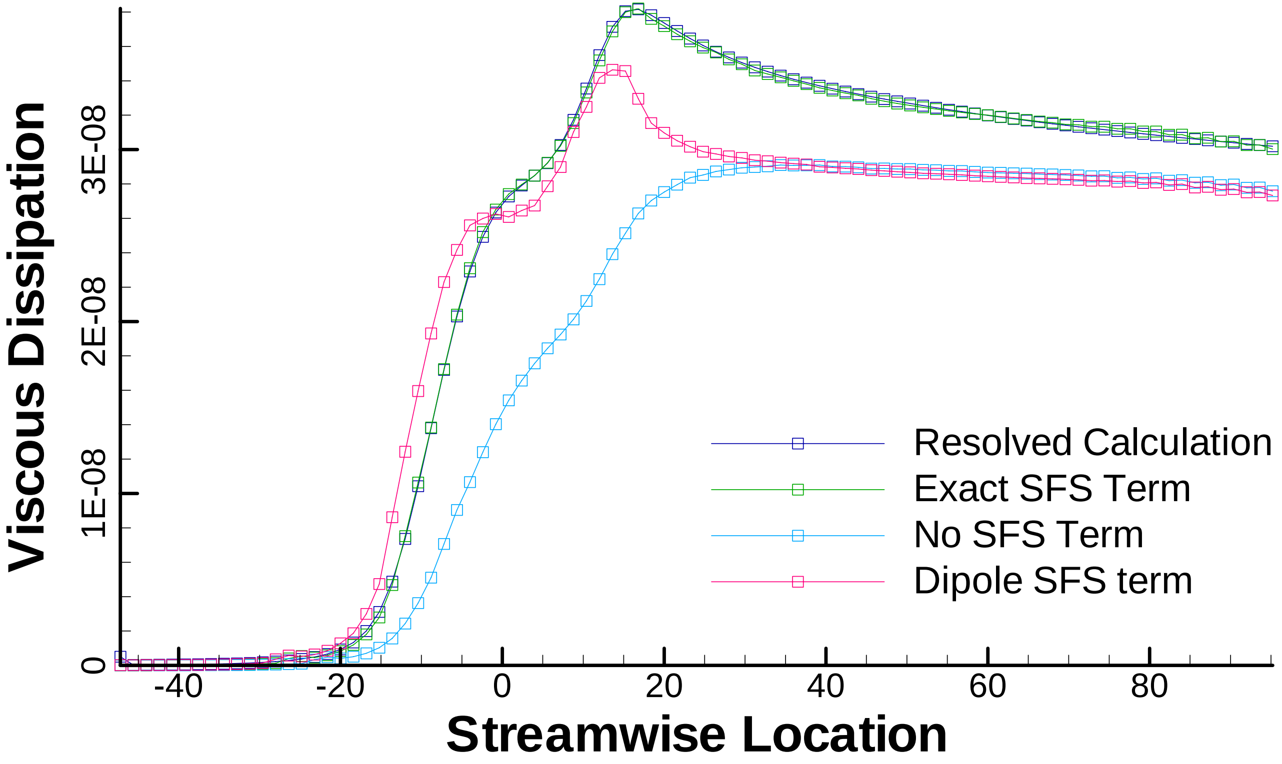
- However, with only the surface forces (as in a point-force model), the dissipation is underpredicted.

Results: $r_{\text{filter}} = 4.0 D_{\text{particle}}$



- Even a simple point-dipole SFS model produces a significant improvement.
- The wake region is not affected by a near-particle model.

Results: $r_{\textbf{filter}} = 16.0D_{\textbf{particle}}$



- For this much smaller particle, the point-force dissipation is better but still underpredicted.

Summary and Conclusions

- We have derived LES-compatible equations for particle-laden flows by spatially filtering the conservation equations for an embedded subdomain.
- These equations can be used to develop models for typical spherical particles, without relying on small-particle assumptions.
- For Re=100 spheres, the subfilter-scale convection term is significant for even fairly small particles, and neglecting it causes underprediction of the viscous dissipation near the particle.
- A very basic point-dipole model for the subfilter-scale convection term provides a significant improvement in the calculated dissipation.
- This method provides a clear path to further improvements in particle models.

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